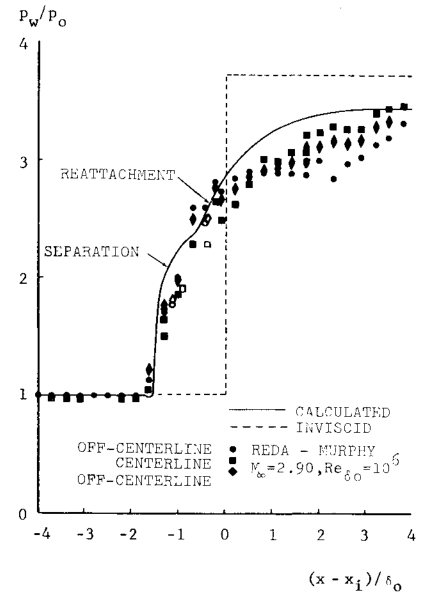

Fig. 1 Comparison of calculated surface-pressure distribution with experimental data; $M_\infty = 2.96$ $\theta = 9.87°$; $Re_{\delta_0} = 2.49 \times 10^5$; open symbols denote pressures measured at separation and reattachment.

All Reda's comments regarding solution accuracy for "quantities indicative of the over-all scale of the interaction region" are qualitative. It is instructive to quantify the actual differences; the level of accuracy is then seen to be quite good.

Pressure Rise to Reattachment

As already noted, p_R/p_0 is within 5% of the Reda-Murphy off-centerline value; more importantly, the computed surface-pressure distribution is within 10% of measured values throughout the interaction region when all Reda-Murphy data are considered.

Upstream Influence Length

Since publishing the Note, I have found that numerical smearing of the incident shock in the freestream caused uncertainty in the location of x_i, and hence x_0, of about $0.15\delta_0$. Reda's Fig. 2 shows a difference between computed and measured $(x_i - x_0)/\delta_0$ of about twice this amount, so that the numerical error accounts for much of the difference between measured and calculated x_0.

Separation Bubble Length

The computed bubble should be larger than the measured bubble because the Reynolds number, Re_{δ_0}, in the numerical flowfield is 2.5×10^5 compared to $Re_{\delta_0} = 1.0 \times 10^6$ for the Reda-Murphy field. That is, Roshko and Thomke[3] have found for Mach 2.95 compression-corner flows that increasing Re_{δ_0} beyond about 10^6 increases the flow's resistance to separation with an attendant reduction in the extent of the separated region. Further verification of this claim is provided by results of two Saffman-model shock-wave boundary-layer interaction flowfields[4] computed since publication of my Note, for a freestream flow deflection angle, θ, of 12.75°; Re_{δ_0} was 2.5×10^5 in one calculation and 1.0×10^6 in the other. The calculations verify that $(x_R - x_s)/\delta_0$ decreases appreciably when Re_{δ_0} is increased from 2.5×10^5 to 1.0×10^6; specifically, the computed separation-bubble length decreases from $3.87\delta_0$ for the lower Re_{δ_0} to $2.81\delta_0$ for $Re_{\delta_0} = 1.0 \times 10^6$.

Sonic Line Angle

The points shown by Reda (Fig. 4) are of doubtful sufficiency to determine the angle at which the sonic line leaves the wall, so that this comparison may have little meaning; data points at $(x - x_i)/\delta_0 = -1.0$ and/or -1.75, for example, would be needed to verify the sketched linear fit to the data. Interestingly, for the $\theta = 12.75°$, $Re_{\delta_0} = 1.0 \times 10^6$ calculation mentioned previously, the computed sonic line leaves the wall at an angle almost identical to that shown in Reda's Fig. 4 when $\alpha = \theta = 13°$, a curve for which sufficient data are available to determine the initial sonic line inclination.

Absolute Displacement of Sonic Line

In light of the preceding comments regarding separation bubble size, the sonic line should be expected to be farther from the wall in the lower-Reynolds-number numerical flowfield than in the experimental flowfield.

In conclusion, the results presented in my Note were not intended to provide a definitive test of the Saffman turbulence model. Rather, the Note only intended to relate to the fluid mechanics community, prior to publication of a more detailed paper, results of a first of its kind calculation of shock-induced turbulent boundary-layer separation and its promising results. The stronger-shock calculations mentioned earlier in this reply do provide a much better test of the turbulence model since both shock strength and Reynolds number more closely match those of the Reda-Murphy 13° freestream flow-deflection interaction.

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Comment on "Evaluation of Preston Tube Calibration Equations in Supersonic Flow"

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DATA are presented by Allen[1] (documented and notation explained in Ref. 2) for Preston tubes in constant-pressure adiabatic-wall supersonic boundary layers at freestream Mach numbers M_e between 2.0 and 4.6 and Reynolds numbers based on tube diameter and friction velocity, $u_\tau d/\nu_w$, between 6 and 5000. The data cover a greater range than previous measurements (referenced by Allen) and appear to be of good accuracy except at small $u_\tau d/\nu_w$. Unfortunately, the empirical "intermediate temperature" concept used in Allen's preferred data correlation (Ref. 1, Fig. 4) is incompatible with the law-of-the-wall concept on which the Preston tube relies: briefly, the intermediate temperature depends on freestream conditions while the Preston tube

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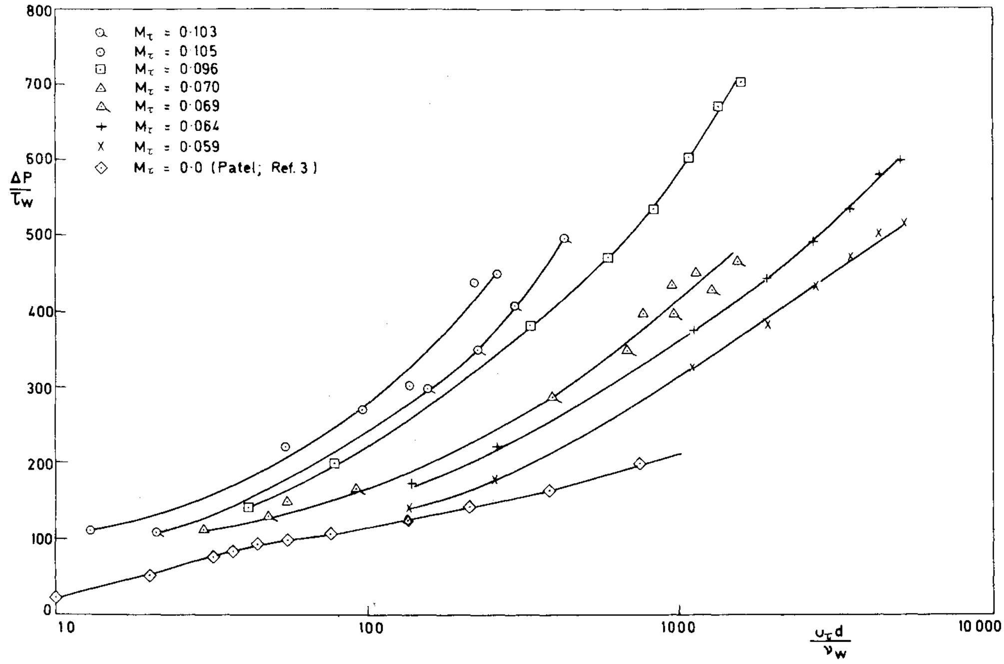

Fig. 1 Allen's results.[1]

calibration depends only on conditions at the wall. Errors are likely to be significant in strong pressure gradients where the skin-friction coefficient differs significantly from that in constant-pressure flow and where the Preston tube is most valuable.

An obvious extension of Preston's dimensional analysis to compressible adiabatic-wall flow shows that the low-speed calibration, most simply written as

$$\Delta p/\tau_w = f_o(u_\tau d/\nu) \tag{1}$$

becomes

$$\Delta p/\tau_w = f_1(u_\tau d/\nu_w, u_\tau/a_w) \tag{2}$$

where the "friction Mach number" $u_\tau/a_w = [\tau_w/(\gamma p)]^{1/2} \equiv M_\tau$, based on the speed of sound at the wall, is the correct law-of-the-wall parameter for compressibility effects. All fluid properties are evaluated at the wall. Several other arrangements of the variables are possible: the form

$$\frac{\Delta p}{\tau_w} = f_o\left(\frac{u_\tau d}{\nu_w}\right) + f_c\left(\frac{u_\tau d}{\nu_w}, \frac{u_\tau}{a_w}\right) \tag{3}$$

splits the calibration into a low-speed part, f_o, which can be chosen as the well-established function given by Patel,[3] and a compressibility correction f_c.

Figure 1 shows Allen's data plotted as $\Delta p/\tau_w$ against $u_\tau d/\nu_w$ for various values of $M_\tau \equiv u_\tau/a_w$. Points with $d/\delta > 0.2$ should be ignored: one cannot expect law-of-the-wall arguments to be valid for such large tubes and in strong pressure gradients the permissible size of tube is even smaller.[3] The curves in Fig. 1 are *hand-drawn fits to the data.* Figure 2 shows a crossplot of the data, using these hand-drawn curves and ignoring points below $u_\tau d/\nu_w = 50$, which appear rather inconsistent in Fig. 1. The curves in Fig. 2 are *from the formula*

$$\Delta p/\tau_w = 96 + 60\log_{10}(u_\tau d/50\nu_w) + 23.7\,[\log_{10}(u_\tau d/50\nu_w)]^2 + 10^4 M_\tau^{\,2}[(u_\tau d/\nu_w)^{0.26} - 2.0] \tag{4}$$

in which the first three terms are a fit to Patel's calibration, accurate to within 2% for $50 < u_\tau d/\nu_w < 1000$, and the last term is the compressibility correction which gives the best fit to Allen's data. *Any* compressibility correction varies as (Mach number)^2 for small Mach number: since in constant-pressure adiabatic boundary layers M_τ becomes roughly constant at about 0.15 as the freestream Mach number tends to infinity, Eq. (4) may be valid at much higher freestream Mach numbers than are covered by Allen's data, in which M_τ was already as large as 0.1. Equation (4) can also be taken as a valid interpolation between Patel's data (at $M_\tau = 0$) and Allen's. The calibration has been left in an implicit form, rather than extracting an explicit formula for τ_w in terms of Δp and the absolute pressure p, for ease of extension when more data appear. To find τ_w, given Δp, p, and ν_w, guess $\Delta p/\tau_w$ (250, say) and insert the resulting value of τ_w in the right-hand side of Eq. (4) to get a new value of $\Delta p/\tau_w$: under-relax by taking the average of the old and new values of $\Delta p/\tau_w$ as input to the next iteration; 4–5 iterations give 1% accuracy. Fuller details, and a short computer program for solution from input in any one of three choices of units, are given in Ref. 4.

More recent work by Brederode at Imperial College suggests that Patel's Preston tube calibration gives values of τ_w at least 2% lower than those deduced from logarithmic velocity profiles using Coles'[5] law of the wall. If one believes the latter rather than the former, the first three coefficients in Eq. (4) should all be reduced by 2% and (since Allen's data are not affected) the empirical constant of -2.0 in Eq. (4) should be changed to about -1.96. However, even at low speeds these

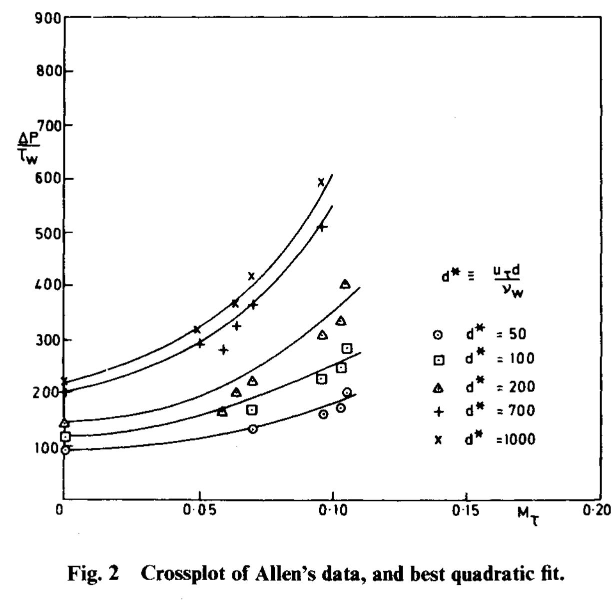

Fig. 2 Crossplot of Allen's data, and best quadratic fit.

effects are at the limit of observational accuracy and can be neglected in the present context.

Finally, it must be emphasized that this work, like that of Ref. 1, applies only to adiabatic-wall boundary layers; in the presence of a heat-transfer rate Q_w, Eq. (2) contains the additional parameter $Q_w/(\rho u_\tau^3)$, and the intermediate-temperature correlation is even more suspect. We cannot confidently extend Eq. (4), or any other formula, without measurements of skin friction in the presence of heat transfer.

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Reply by Author to P. Bradshaw and K. Unsworth

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I WISH to thank Bradshaw and Unsworth for their interest in my paper[1] and for using my data in developing their calibration equation. In order to assess their equation on the same basis as the ones evaluated in my paper, I have compared the skin friction determined from their equation [Eq. (4) of their paper] with measured skin friction in a manner similar to the evaluations performed in my Synoptic, and would like to present my findings.

First of all, a few words about the intermediate temperature concept. I recognize that this concept is not theoretically exact; however, it has proven very useful in obtaining engineering estimates of compressibility effects in zero pressure gradient boundary-layer prediction techniques. Because of its inexact theoretical basis, the usefulness of the intermediate temperature concept depends on its demonstrated validity over a wide range of test conditions, which was, indeed, the reason for generating the large volume of data in my study. Because my calibration equation (see Fig. 4 of my Synoptic) is based on the intermediate temperature concept, I recognize that it is probably invalid in strong pressure gradient flows; however, I feel it has a demonstrated validity in zero pressure gradient flow as shown in Fig. 5 of my Synoptic.

Figure 1 was prepared to show how the skin friction determined from the Bradshaw-Unsworth equation compares with measured skin friction for the three independent sets of supersonic calibration data used to evaluate the various equations in my Synoptic. The Bradshaw-Unsworth equation provides a very good fit to my data—which is to be expected since it was derived from my data—whereas my equation, being derived from all three sets of data, provides a slightly better fit to the total data available, as seen in Fig. 5 of my Synoptic. The differences in accuracy between the two equations, however, are relatively minor.

One of the principal conclusions of my paper was that valid results were obtained with large diameter tubes ($D/\delta > 0.2$) even though, as Bradshaw and Unsworth state, "one cannot expect law-of-the-wall arguments to be valid for such large tubes...." Figure 1 shows that the Bradshaw-Unsworth equation also gives large-tube results which compare favorably with the smaller-tube results. The reason that these large tubes give valid results can be seen in Fig. 2 in which measured velocity profiles obtained with large and small tubes are compared to the true profile (obtained in Ref. 4 by extrapolation to $D = 0$). Although the traverse of the large tube through the boundary layer produced a severe distortion of the measured profile compared to the true

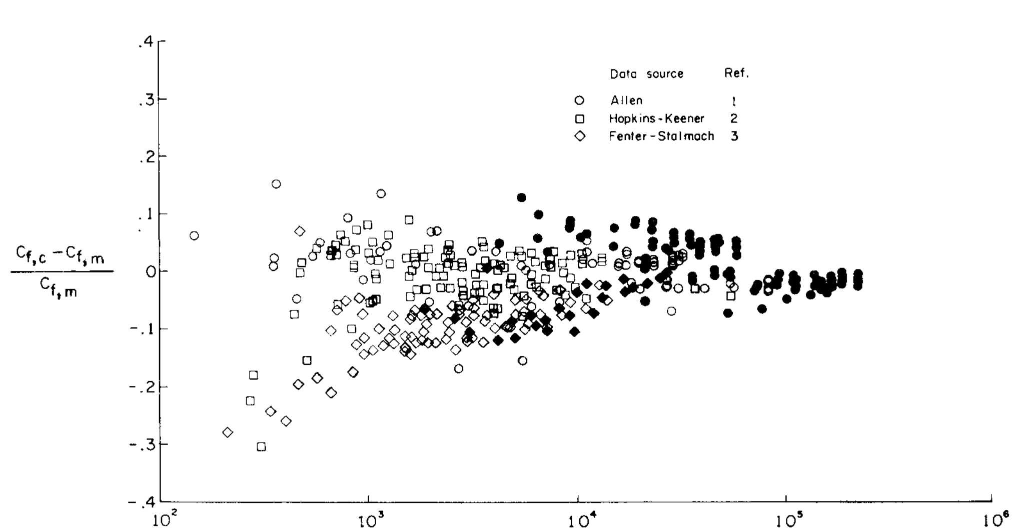

Fig. 1 Comparison between measured friction and friction determined from Bradshaw-Unsworth equation. Solid symbols denote data for $D/\delta > 0.2$.

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